



SCHEMA AS VISUAL REPRESENTATION IN SOLVING CALCULUS WORD PROBLEMS: A LITERATURE REVIEW

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ABSTRACT

The aim of this article is to provide literature review on why and how schema is used to help students solve calculus word problem. It starts with the description of typical difficulties found in common word problem, calculus, and both. Then, it continues with the explanation of what is schema and its advantages. The examples of promoting schema in classroom are also analyzed from several literatures. The strengths of schema are discussed yet some considerations on promoting it are also analyzed.

KEYWORDS: calculus, word problem, schema, mathematics.

INTRODUCTION:

Word problem has been notoriously challenging for its complex solving process. It requires students to pass through several steps, i.e. reading the problem, understanding the problem, transforming information into mathematical model, processing the model, interpreting the result back to problem context, and check the result (Reys, Lindquist, Lambdin, & Smith, 2013; Ryan & Williams, 2007; Verschaffel, van Dooren, Greer, & Mukhopadhyay, 2010). Many investigations have been done to provide a systematic way to help students to solve word problems, and one that emerged quite massively is the use of visual representation (e.g. Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Hegarty & Kozhevnikov, 1999; Jitendra et al., 2009; Powell, 2011; Presmeg, 2006; Stylianou & Silver, 2009).

The use of visual representation still has been considered a promising way to "bridge" wordy texts of word problems into mathematical solution. The use of one of visual representation type, i.e. schema, has been promoted for advanced mathematics like calculus, as it demands more cognitive process, it expects more tool as a help (Baker, Cooley, Trigueros, & Trigueros, 2000; Bremigan, 2005).

The aim of this article is to provide description of the characteristics of challenges faced by students related to calculus word problems. After knowing those characteristics, this article will analyze literatures to explain the advantage of using schema and how it should be taught to students during word problem solving.

Calculus and Word Problem Difficulty:

Calculus covers several subtopics such as limit, derivative, and integral. The mastery of these topics is supported by the mastery of their prerequisites such as, among others, the understanding of function, manipulating algebraic expressions, trigonometric manipulation, and geometry (Sofronas et al., 2011). The topic is very demanding by itself, let alone when it is put in the word problems.

Calculus word problems usually are constructed by first designing the context and information that are intended for students to find the representative function. Depends on the objective of the problem, students then should use the function they found to reach the objective, i.e. finding the solution to the word problem. For example, in a study, students were given the following problem.

The cost of running a heavy truck at a constant velocity of v km/h is estimated to be $4 + v^2/200$ dollars per hour. Show that to minimize the total cost of a journey of 100 km in the truck at constant velocity the truck should run approximately 28 km/h (Klymchuk, Zverkova, Gruenwald, & Sauerbier, 2010, p. 83).

The context of this problem is about total cost of a journey. In order to do the problem, students should construct the function of total cost by making use of the given information. The goal of this problem is to show the minimum total cost when the truck velocity is 28 km/h. It means that students should find the value being substituted to the function such that the function will get its minimum value. However, the problem did not seem to be easy for students, as only 10 out of 197 participants succeeded to give the correct function (Klymchuk et al., 2010).

The problem required students to use the understanding that total cost would be the cost per hour multiplied by the length of time while the length of time is the distance (100 km) divided by the velocity. This concept of total cost should be understood by students so that they can determine the function. From this problem, it was found that students could not understand the vocabulary, could not

understand the objective of the problem, or got confused by the sentences. This finding was in line with the finding of the study by Stillman (2004) where students found it difficult to point out important words to solve the problem. This was due to their lack of understanding of the technical language and textual structure of the word problem. It was shown that the difficulty, rather than came from the understanding of calculus concept, came from the understanding of the textual information instead. This leads to the urgency of a particular way to help students organize the information given in the problem.

Schema For Solving Calculus Word Problem:

Schema, a framework, diagram, or plan to organize information (Marshall, 1995), can be one of the ways. Schema, in the process of solving word problems, is used to organize information by depicting the relation among concepts into a structured concept. In other words, schema can be the connector between the word problem and students' activity in solving it.

In relation to word problems, Powell (2011), describe steps in using schema in solving word problems, i.e. reading the problem, identifying and categorize information into relevant or irrelevant information, making the schema by putting the relevant information or its equal representations, and solve the problem. The whole process is formed into a schema in which all relevant information to solve the problem exist. The flow of solving word problem using schema by Powell (2011) is given in Figure 1.

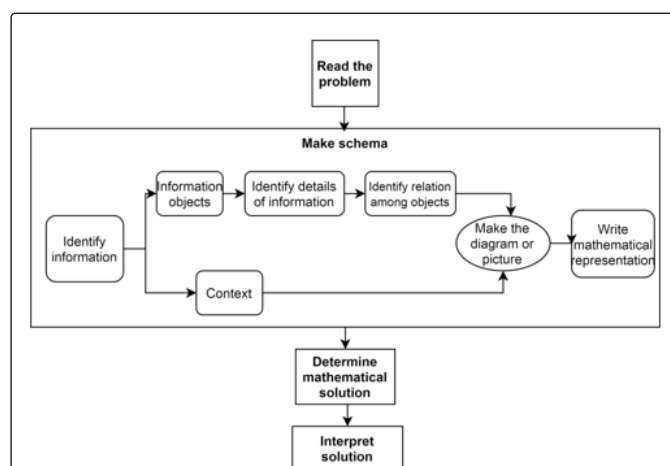


Figure 1: Solving word problem using schema, modified from Powell (2011)

Schema is not merely a visual representation of a problem, but also a tool to picture the relations among relevant information (Hegarty & Kozhevnikov, 1999). Students are helped to choose appropriate visual representation from textual information in the problem. Besides, the complexity of the language structure (syntax) of the problem can be reduced by making a schema. The making of schema in solving word problem is proven to be effective in reducing working memory of the brain during the process of understanding complex syntax in the word problem (Jitendra et al., 2009). Solving word problem using schema can also help students to solve efficiently and at the same time train them to solve it in the future (Skemp, 1987). This is because schema can be an information

decomposer as well as a tool to solve them.

Promoting the use of schema can be done by allocating more time for students to create and discuss schema during their process of solving problem. The study of Bremigan (2005) gave an example of how schema was used to solve word problem of derivative. This study investigated how students used schema to summarize information written in the word problem. The word problem given in this study was as follow.

An oil storage tank has the shape created by revolving the curve $y = \frac{9}{625}x^4$ from $x=0$ to $x=5$ about the y-axis, where x and y are measured in feet. Oil weighing 50 pounds per cubic foot flowed into an initially empty tank at a constant rate of 8 cubic feet per minute. When the depth of the oil reached 6 feet, the flow stopped. Let h be the depth, in feet, of oil in the tank. How fast was the depth of the oil in the tank increasing when $h = 4$? (Indicate units of measure) (Bremigan, 2005, p. 258)

From this problem, students were guided to construct schema to organize the information given (the final schema and solving process were given in Figure 2). In this problem, students first were guided to transform the given equation into a visual representation of the oil storage. At first, students drew a simple graph representing the equation (top left part), then teacher could guide them to complete it into a whole oil storage picture with notes in the picture. The schema's role is visualizing the equation and displaying relevant information. By creating this schema, it is hoped that students could process the information in the schema into solution.

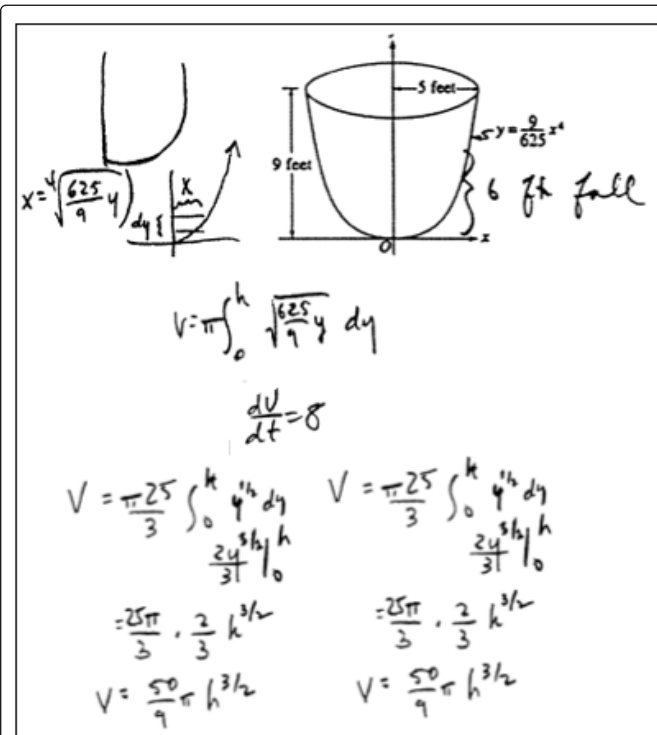


Figure 2: Schema on oil storage problem (Bremigan, 2005, p.258)

Regarding visual representation, Eisenberg (2002) gave note to consider for teachers. In his study, students usually were divided into those who prefer analytical (non-visual) way and those who prefer visual way. However, in mathematics topic involving function, he emphasized the concern on failure mostly faced by students who prefer non-visual analytical process. Thus, teacher is supposed to be wise in promoting schema, as it also deals with student's cognitive style.

Conclusions:

Schema is a promising tool to help students solve word problems of calculus. It helps students to deal with common difficulties faced by students and also with characteristics of calculus as advanced mathematics. It helps students to decompose the information given, acts as bridge between textual information and mathematical process, and reduces students' working memory.

Promoting schema requires effort and knowledge of teachers about its benefits and when it should be used. The consideration of students' solving style should be done while giving more time in class to discuss how and when schema will be useful.

Acknowledgments:

This research was supported by the Indonesia Endowment Fund for Education (Lembaga Pengelola Dana Pendidikan, LPDP).

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